A COMPARISON OF SOME LANCHESTER MODELS OF THE SKIRMISH

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THESIS

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ABSTRACT

Four Lanchester-type models are examined to investigate the hypothetical attrition process in skirmishes between ground forces. Analytic solutions are developed to Lanchester-type equations of warfare for combat between two homogeneous forces in the following circumstances:

(1.) linear-law attrition process,

(2.) square-law attrition process with constant attrition-rate coefficients,

(3.) square-law attrition process with linearly-decreasing, time varying attrition rates,

(4.) square-law attrition process with exponentially-decreasing, time varying attrition rates.

The above models are applied to specific combat scenarios typical of a counterinsurgency environment. The adequacy of such models as defense planning guides is discussed through a critical examination of the assumptions (both explicit and implicit) which lead to them.

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I. INTRODUCTION

Classical Lanchester theory provides two means of modeling attrition resulting from combat between two homogeneous forces. Each is comprised of a pair of differential equations, and each equation is associated with one of the opposing combatants. One model, known popularly as the Lanchester square law, essentially states that one's losses are directly proportional to the number of enemy encountered. It has been postulated that this model describes combat losses due to aimed (rather than area) fire. On the other hand, the so-called Lanchester linear law describes the attrition process due to area fire, i.e., fire simply directed at the area thought to be occupied by the enemy, rather than directly at the individual enemy units. This latter model states that such attrition is proportional to both the number of enemy encountered and the size of one's own force. Bonder [1] provides a succinct and thorough discussion and comparison of these two basic Lanchester models.

Lanchester-type equations are widely employed in computerized war games and other simulation studies for defense planning. Such games and models enjoy wide usage as aids in recommending future force levels, predicting procurement needs and developing tactical and strategic policies. In addition, there are other, less tangible but no less important benefits in the area of gaining new insight into various combat scenarios. These benefits are acquired through clever

manipulation of the classical Lanchester models to include extensions of the theory which consider an increased number of combat parameters and thus achieve a higher degree of realism. To this end, many varied combat situations have been modeled via Lanchester theory. One extremely common tactical circumstance has, however, escaped much of the scrutiny experienced by other more dramatic scenarios, and this is the skirmish or common firefight.

Deitchman [2] and Schaffer [3] both examined the skirmish in their papers on the modeling of guerrilla engagements but their definitions of the action were extremely restrictive in a tactical sense and indeed at odds with the current prevailing usage of the term within the defense establishment.

Webster defines the word "skirmish" as "... a brief fight or encounter carried on between small groups, usually part of a battle or war..." Elaborating a bit, the term has come to mean a small-unit (forces of 300 or fewer men), random encounter, as opposed to a determined assault on a specified objective. As an illustrative example, consider two units -- more often than not patrols -- moving through a given tactical area. At some point in time, the units might make contact, the resulting combat being a firefight or skirmish. The major factor which seems to distinguish the skirmish from other forms of combat is the fact that the engagement is the result of chance encounter rather than prior planning. While it is true that last-minute decisions by the commanders of the forces involved might well result in

combat being either joined or avoided, it is likewise true that in many cases, the engagement is precipitated in a manner which is independent of the wishes or judgement of both commanders.

There are other distinguishing characteristics of the skirmish. These include the all-too-often lack of any clear-cut tactical decision, i.e., the failure of either side to achieve a victory or suffer a defeat in the accepted sense of the terms for conventional rectillinear warfare. Indeed, the skirmish derives its almost strategic importance from the fact that oftentimes it is the overall attrition of many skirmishes over a wide operational front that contributes heavily to military success or failure. Thus, it becomes a simple matter to see -- in terms of the skirmish -- how a military force can lose a war without losing a battle.

This latter consideration serves as an indication of the importance of the skirmish as a tactical occurrence worthy of attention by the military operations analyst. To underscore this importance, it is well to note that recent military experience in Southeast Asia has served to emphasize the high incidence and importance of the firefight in so-called "wars of liberation." Theaters which witness the extensive employment of guerrilla tactics, search-and-destroy operations and vigorous patrolling will necessarily experience extensive skirmishing -- perhaps even to the virtual exclusion of all other forms of ground combat.

The research task chosen by the writer and described in this thesis was the construction of Lanchester models of the

skirmish and a critical examination of the adequacy of their description of the combat process. Along these lines, the applicable assumptions underlying various aspects of Lanchester theory were examined in the light of the skirmish. Further, analytic solutions to the equations describing the attrition processes were obtained and examined with a view towards military plausibility. This latter task was accomplished by means of computer programs, utilizing hypothetical combat parameters and printing out the force levels for skirmishes of various durations. The attempt was made to progress from the basic, classical, constant-coefficient Lanchester model to the more sophisticated and more complex case where attrition coefficients are decreasing exponential functions of time.

II. THE AIMED-FIRE, CONSTANT-COEFFICIENT MODEL

A. SCENARIO AND ASSUMPTIONS

In describing a skirmish by means of Lanchester-type equations, one of the first questions which arose was: square law or linear law? In answering this letter question, it first became necessary to inquire: aimed fire or area fire? In the author's experience, many a young combat rifleman has claimed that <u>his</u> fire was aimed. On the other hand, there exists an equally convincing, albeit more cynical argument posed by older, more seasoned combat leaders, who cited shock, confusion and inexperience in claiming that combatants in firefights are more inclined to simply blast away in the general direction of the enemy rather than taking careful aim.

In modeling the engagement under the assumption that the former claim is true -- i.e., that aimed fire predominates -- it was necessary to use the square law, so-called because of the form of its solution. Square law attrition, then, was modeled using the following differential equations:

$$\frac{dX}{dt} = - \alpha Y, \tag{1}$$

and

$$\frac{dY}{dt} = - \beta X \tag{2}$$

where:

X and Y are the average strength of the X and Y forces, respectively;

α = the constant rate at which a Y-force individual kills individuals of the X-force;

β = the constant rate at which an X-force individual kills individuals of the Y-force.

Hence, dX/dt and dY/dt represent the attrition with respect to time of the X and Y forces, respectively.

α and β are commonly referred to as attrition coefficients, and measure, in effect, the loss-inflicting effectiveness of the forces engaged. Deitchman has postulated that such attrition coefficients are structured as follows:[2]

$$\alpha = r_y \, p_{ky} \tag{3}$$

where:

r_y = the rate of fire of each of Y's engaged weapons systems (where homogeneity is assumed) and

p_{ky} = the single-shot kill probability of each of Y's weapons systems.

The skirmish modeled by these equations was visualized by the author as a conflict between two small units -- specifically, company-sized (300 men) or smaller. Such an action would last from one to ten minutes, and would be characterized by relatively few casualties on either side.

The assumptions underlying the model of a battle of this sort are:

(a) Two forces, composed of identical individuals, are engaged;

(b) each unit on either side is within range of all the weapons of the opposing force;

(c) there is no serial correlation between rounds fired by either unit;

(d) the effect on the target array of any shot fired is independent of the effect of any other shots fired at it;

(e) upon destroying a target, the firing unit immediately shifts his fire to a fresh "live" target;

(f) fire -- at all times -- is uniformly distributed over remaining live targets. [4]

The last two assumptions imply directly that the action is characterized by the employment of aimed fire by both sides. The debate surrounding this contention was commented on above.

B. NUMERICAL COMPUTATIONS

In testing such a model with hypothetical data, it was necessary to utilize a solution in terms of X(t) and Y(t), i.e., strength of the X and Y forces after participating in a skirmish for a time t. In the case of the square-law attrition model, a closed-form solution was available, to wit:

$$X(t) = X_0 \text{Cosh}(\sqrt{\alpha\beta}\ t) - Y_0\sqrt{\alpha/\beta}\ \text{Sinh}(\sqrt{\alpha\beta}\ t), \quad (4)$$

and

$$Y(t) = Y_0 \text{Cosh}(\sqrt{\alpha\beta}\ t) - X_0\sqrt{\beta/\alpha}\ \text{Sinh}(\sqrt{\alpha\beta}\ t), \quad (5)$$

where:

X_0 and Y_0 represent the initial strengths of the X and Y forces, respectively, and

t = time.

Arriving at figures to represent the required parameters was an arbitrary process. Force sizes of 100 and 200 men were chosen. Skirmish durations varied from 100 to 600 seconds and attrition was computed in terms of men/second. The

individual rates of fire used were .167 rounds/man-second and .084 rounds/man-second, or approximately ten and five rounds per minute per man, respectively. These latter figures represent feasible -- and therefore plausible -- figures and nothing more. Their use enables an examination of the behavior of the attrition process, such behavior exhibiting similar characteristics whatever the actual numerical value assigned to the rates of fire.

Single-shot kill probabilities of .001 and .002 were used. While these values are plausible (or perhaps optimistic), their purpose was, as with the rates of fire, to aid in demonstrating the general behavior of the attrition process and not to be used as a means of predicting actual attrition figures.

Using such figures resulted in constant attrition coefficients on the order of .00167 seconds^{-1}.

A small FORTRAN IV (G) computer program was written to assign parameter values and solve for attrition data for skirmishes of various durations. The program appears in the appropriate section at the end of the text. Results of the computations are listed in Table One and a typical case, showing the attrition process' behavior graphically, is portrayed in Figure One.

C. RESULTS AND CONCLUSIONS

In all cases examined, the attrition process was a nearly linear function of time, i.e., force strength appeared to decrease almost linearly with time. Clearly, a significantly greater rate of fire or a significantly greater

TABLE ONE

SQUARE LAW ATTRITION DATA -- CONSTANT COEFFICIENTS

INITIAL STRENGTHS (MEN)		RATE OF FIRE (rds/sec)		KILL PROBABILITY		TIME	FINAL STRENGTHS (MEN)	
X	Y	X	Y	X	Y	(sec)	X	Y
200	200	.167	.167	.001	.002	100	193	197
200	200	.167	.167	.001	.002	200	187	194
200	200	.167	.167	.001	.002	300	180	190
200	200	.167	.167	.001	.002	400	174	188
200	200	.167	.167	.001	.002	500	168	185
200	200	.167	.167	.001	.002	600	162	182
200	200	.167	.084	.001	.001	100	198	197
200	200	.167	.084	.001	.001	200	197	193
200	200	.167	.084	.001	.001	300	195	190
200	200	.167	.084	.001	.001	400	194	187
200	200	.167	.084	.001	.001	500	192	184
200	200	.167	.084	.001	.001	600	190	180
200	100	.167	.167	.001	.001	100	198	97
200	100	.167	.167	.001	.001	200	197	93
200	100	.167	.167	.001	.001	300	195	90
200	100	.167	.167	.001	.001	400	194	87
200	100	.167	.167	.001	.001	500	192	84
200	100	.167	.167	.001	.001	600	191	80
200	100	.084	.167	.001	.001	100	198	98
200	100	.084	.167	.001	.001	200	197	97
200	100	.084	.167	.001	.001	300	195	95
200	100	.084	.167	.001	.001	400	194	93
200	100	.084	.167	.001	.001	500	192	92
200	100	.084	.167	.001	.001	600	190	90
200	100	.084	.167	.001	.002	100	197	98
200	100	.084	.167	.001	.002	200	193	97
200	100	.084	.167	.001	.002	300	190	95
200	100	.084	.167	.001	.002	400	187	94
200	100	.084	.167	.001	.002	500	184	92
200	100	.084	.167	.001	.002	600	181	90

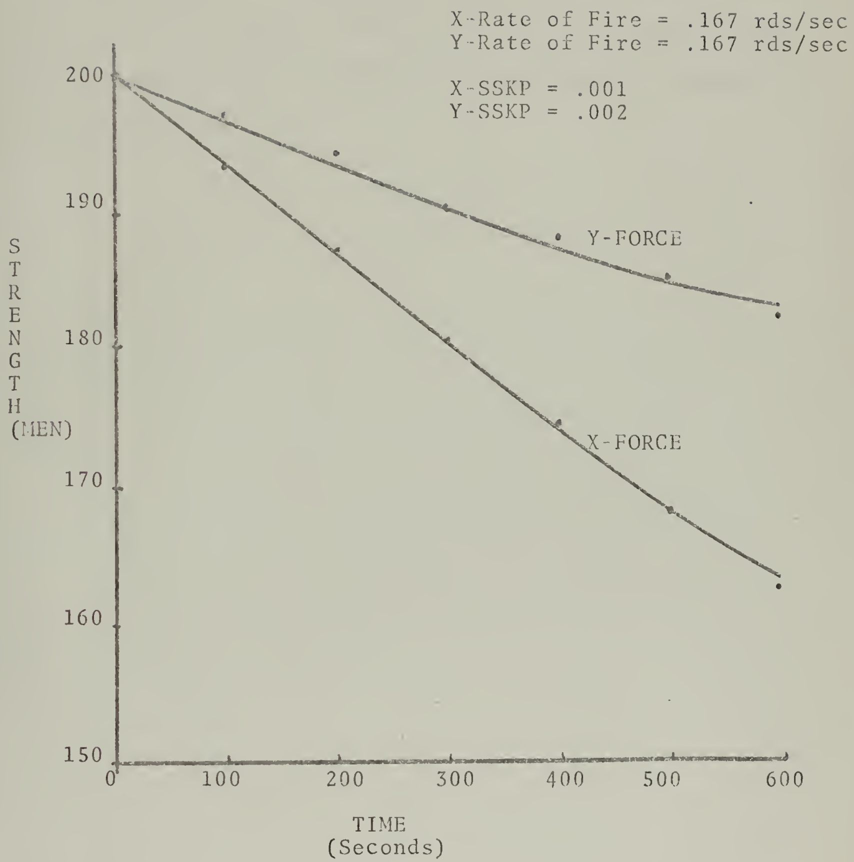

FIGURE ONE

TYPICAL SQUARE-LAW ATTRITION -- CONSTANT COEFFICIENTS

accuracy (manifested in the single-shot kill probability) appear to give a force a decided advantage in an engagement such as the one modeled here. To a lesser extent, an initial advantage in force size proves to be advantageous, as might be expected. This latter advantage, however, could be overcome by the smaller force having a great rate of fire and/or a greater kill probability, provided the engagement lasts long enough.

III. THE AREA-FIRE, CONSTANT-COEFFICIENT MODEL

A. SCENARIO AND ASSUMPTIONS

The scenario upon which this model was based was very similar to that used for the preceding model. Essentially all that was involved was an allowance for the employment of area fire by both forces, made via a modification of the assumptions as follows:

The first four assumptions [(a) through (d)] remained the same as those used in the square-law model; remaining assumptions are:

(e) each individual is aware of the general area within which his opponent is located, but is not informed (or doesn't care) about the precise consequences of his own fires;

(f) fire from surviving individuals is distributed uniformly over the opposing force's area;

(g) each individual presents the same vulnerable area to the opposing force's fire;

(h) only one hit is required for a kill.

These assumptions serve to yield the attrition model described by the following equations:

$$\frac{dX}{dt} = - AXY, \tag{6}$$

and

$$\frac{dY}{dt} = - BYX, \tag{7}$$

where A and B are attrition coefficients [4].

The assumptions listed above serve to highlight the difference in the nature of the attrition coefficients from their square-law counterparts, especially in the single-shot kill probability component of the respective parameters. As an example, the constant coefficient A in equation (6) is indeed the product of the unit or individual rate of fire of the Y-force times the latter's single-shot kill probability. However, this probability, due to the assumptions regarding area fire, is approximated as follows:

$$p_{ky} = a_x/A_x \tag{8}$$

where

a_x = the vulnerable (i.e., exposed) area of each individual of the X-force,

and

A_x = the total area occupied by the X-force (or, more properly, the area into which the Y-force is actually firing). [2]

It can be readily seen that a realistic area-fire scenario can reduce the magnitude of this term considerably, since, for example, normal tactical dispersion in open terrain might precipitate a situation where a_x is on the order of one square foot, being the exposed area of an infantryman (possibly in a prone position or taking advantage of some sort of cover) while A_x, the area occupied by a company-sized X-force, might reach as high as several hundred thousand square feet.

B. NUMERICAL COMPUTATIONS

Bonder [1] developed the closed-form analytic solutions to the pair of differential equations describing linear-law attrition and expressed them as follows:

$$x(t) = \frac{-X_0(\phi-1)}{\exp(-AY_0(\phi-1)t)-\phi} \tag{9}$$

and

$$y(t) = \frac{-Y_0(\phi-1)\exp(-AY_0(\phi-1)t)}{\exp(-AY_0(\phi-1)t)-\phi} \tag{10}$$

where

$$\phi = \frac{BX_0}{AY_0} \tag{11}$$

Another short computer routine was drawn up in order to determine linear law attrition behavior for a set of combat situations analogous to those described by the previously-discussed square-law model. In maintaining this analogy, every effort was made to keep the skirmishes under the respective models the same in all respects except that of the method of fire employed. To this end, force sizes were maintained at similar levels for the commencement of each skirmish. In addition, attrition was examined for five initial situations, each progressing from 100 seconds to a maximum of 600 seconds in duration.

Because of the area-fire consideration under the linear-law model, the author arbitrarily assigned rates of fire values which were exactly twice those used in the aimed-fire model. Further, in order to maintain the attrition figures at an order of magnitude similar to those obtained from the square-law model (at least during the early stages of the skirmish) A_x and A_y, the areas occupied by the X and Y forces

respectively, were assigned values which were at once tactically plausible and at the same time rendered the values of AX_0 and BY_0 close to the values of α and β which were used in the square-law model.

Results of the simulation are listed in Table Two, with a typical case of attrition behavior portrayed graphically in Figure Two.

C. RESULTS AND CONCLUSIONS

Once again, as with the square-law model, the attrition approximated a linear function of time, although the approximation is not as close as that which existed in the previous case. Attrition under the linear-law model paralleled that of the aimed-fire model very closely. Data shown in Table Two and Figure Two are, as mentioned above, representation of cases where the areas occupied by the forces involved were set at a size such that the product of the linear-law coefficient times the opposing force size was approximately equal to the square-law coefficient, i.e., such that

$$AX_0 = \alpha,$$

and

$$BY_0 = \beta.$$

Other test runs of the computer program, utilizing figures for the respective areas which seemed slightly more realistic, resulted in smaller areas, greater kill-probabilities and hence greater attrition figures. This suggests, among other things, that the actual kill probabilities used

TABLE TWO

LINEAR LAW ATTRITION DATA -- CONSTANT COEFFICIENTS

INITIAL STRENGTHS (MEN)		RATE OF FIRE (rds/sec)		AREA OCCUPIED BY (ft^2)		TIME	FINAL STRENGTHS (MEN)	
X	Y	X	Y	X	Y	(sec)	X	Y
200	200	.334	.334	400,000	200,000	100	194	197
200	200	.334	.334	400,000	200,000	200	187	194
200	200	.334	.334	400,000	200,000	300	181	191
200	200	.334	.334	400,000	200,000	400	176	188
200	200	.334	.334	400,000	200,000	500	170	185
200	200	.334	.334	400,000	200,000	600	165	183
200	200	.334	.167	400,000	200,000	100	198	197
200	200	.334	.167	400,000	200,000	200	197	194
200	200	.334	.167	400,000	200,000	300	195	190
200	200	.334	.167	400,000	200,000	400	194	187
200	200	.334	.167	400,000	200,000	500	192	184
200	200	.334	.167	400,000	200,000	600	191	181
200	100	.334	.334	200,000	400,000	100	198	97
200	100	.334	.334	200,000	400,000	200	197	94
200	100	.334	.334	200,000	400,000	300	195	91
200	100	.334	.334	200,000	400,000	400	194	88
200	100	.334	.334	200,000	400,000	500	192	85
200	100	.334	.334	200,000	400,000	600	191	82
200	100	.167	.334	200,000	400,000	100	198	98
200	100	.167	.334	200,000	400,000	200	197	97
200	100	.167	.334	200,000	400,000	300	195	95
200	100	.167	.334	200,000	400,000	400	194	94
200	100	.167	.334	200,000	400,000	500	192	92
200	100	.167	.334	200,000	400,000	600	191	91
199	100	.167	.334	200,000	400,000	100	196	98
199	100	.167	.334	200,000	400,000	200	193	97
199	100	.167	.334	200,000	400,000	300	190	95
199	100	.167	.334	200,000	400,000	400	186	94
199	100	.167	.334	200,000	400,000	500	184	92
199	100	.167	.334	200,000	400,000	600	181	91

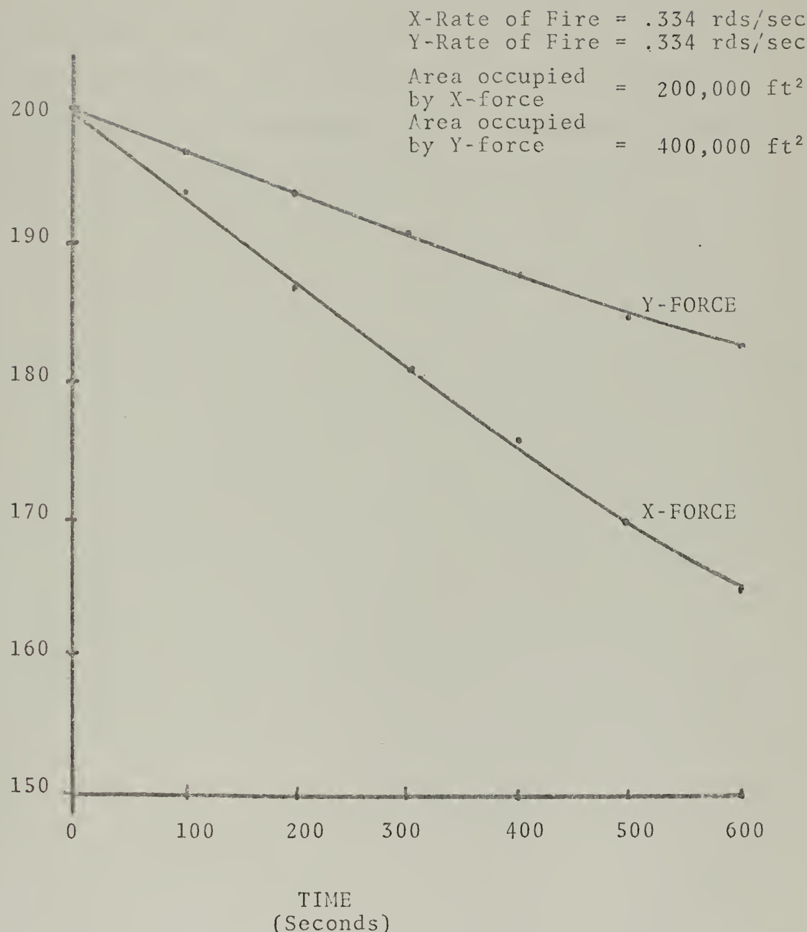

FIGURE TWO

TYPICAL LINEAR-LAW ATTRITION -- CONSTANT COEFFICIENTS

in the aimed-fire model may well have been too high. Further, the difficulties in modeling analogous combat situations where the only difference between the two models is the type of fire employed became quite apparent, at least for cases where numerical results are desired. Suffice it to say at this point, however, that, allowing for the limitations of the existing form of Lanchester theory, both models (square law and linear law) give remarkably similar results.

IV. TWO SQUARE-LAW MODELS WITH TIME-DEPENDENT COEFFICIENTS

Since the original postulation of the Lanchester theory of combat in 1916, several analysts have considered the logical extension of the constant-coefficient models -- models which utilize variable attrition coefficients. Indeed, it is appealing to consider a model of combat where the attrition coefficients -- which are, in effect, simply a measure of one force's effectiveness in killing members of the opposing forces --vary, either as a function of the distance between the opposing forces, or as a function of time. Such considerations are limited in their application for reasons of mathematical tractability. Simple closed-form solutions have been developed for certain restrictive cases of range-dependent and time-dependent coefficients, and the research described here concerned itself solely with the latter since, in the author's experience, most skirmishes seem to occur at a range which remains more or less constant.

On the other hand, the idea of attrition coefficients which vary with time was quite appealing as it seemed to be in accordance with reality. The question of how the coefficients varied was another problem, indeed a problem whose answer is limited somewhat to those cases for which mathematical solutions have been developed. Considered here was the case of the square-law (aimed-fire) model, the equations to which can be written

$$\frac{dX}{dt} = -\alpha(t)Y, \qquad (12)$$

and

$$\frac{dY}{dt} = -\beta(t)X, \tag{13}$$

where $\alpha(t)$ and $\beta(t)$ are attrition coefficients in the same sense as in the earlier model but are now functions of time, rather than constants.

The solutions to the above pair of differential equations are a good deal more complicated than those solutions applicable in the constant-coefficient case. There are, however, some reasonable restrictions which, when placed on $\alpha(t)$ and $\beta(t)$, enable the use of a straightforward closed-form solution to equations (12) and (13)

One useful restriction on the coefficients which yields a tractable and useful closed-form solution was first noted by Taylor, who cited the case where the ratio, $\alpha(t)/\beta(t)$ is a constant, even though the terms by themselves are variable functions of time [5]. When such a case occurs, the explicit solutions for $X(t)$ and $Y(t)$ can be written

$$X(t) = X_0 \operatorname{Cosh}\theta + Y_0\sqrt{K_\alpha/K_\beta}\, \operatorname{Sinh}\theta, \tag{14}$$

and

$$Y(t) = Y_0 \operatorname{Cosh}\theta + X_0\sqrt{K_\alpha/K_\beta}\, \operatorname{Sinh}\theta, \tag{15}$$

where K_α and K_β are constants such that

$$\frac{\alpha(t)}{\beta(t)} = \frac{K_\alpha h(t)}{K_\beta h(t)} = \frac{K_\alpha}{K_\beta}\ , \tag{16}$$

and

$$\theta(t) = \sqrt{K_\alpha K_\beta}\ \int_0^t h(u)\,du. \tag{17}$$

In such a case, it seemed reasonable to examine a hypothetical skirmish where attrition coefficients decreased in

time, maintaining all the while a constant ratio as above. Recalling, for example, that the attrition coefficient $\alpha(t)$ is the product of the individual rate of fire of the Y-force times the Y-force's single-shot kill probability, one can write

$$\alpha(t) = r_y(t)\ p_{ky}.$$

In such a case, single-shot probability is again treated as a constant, while the rate of fire is allowed to vary with time.

Further, in the case of the skirmish, it is readily possible to extend this concept to the point where the rate of fire of a combatant is assumed to be a decreasing function of time. Rationale for this approach is centered in the infantry experience of the author, wherein an engagement of this type was typically characterized initially by a high degree of excitement on the part of the individual combatants, resulting in a high initial rate of fire. As a degree of tactical integrity and order restored, fire discipline increases and fire becomes slower and more controlled. (Note: In addition to a decreasing rate of fire, one might logically expect an increasing accuracy, manifested in a single-shot kill probability which is an increasing function of time. Because of the complexity and indefineable nature of such an assumption, inspection here was limited to the simpler case, wherein kill probability is held constant.)

A. LINEAR ATTRITION COEFFICIENTS

1. Scenario and Assumptions

The simplest case was considered first. This was where the rate of fire was linear and where kill probability was held constant. Standard Lanchester-theory assumptions for aimed fire, as cited above, apply in this case. In addition, rate of fire was treated as a function of time which decreases to zero at the end of the skirmish and which can be written

$$r(t) = mt + b,$$

where

$$m = - \frac{\text{Initial Rate of Fire}}{\text{Duration of Skirmish}},$$

$$b = \text{Initial Rate of Fire},$$

and

$$t = \text{Time}.$$

Such a linearly-decreasing rate of fire is graphed in Figure Three. Once such a linear rate of fire is assumed, it becomes a simple matter to write the Y-force's attrition coefficient as

$$\alpha(t) = p_{ky}(mt + b). \tag{18}$$

Further, if the assumption is made that the rates of fire of both opposing forces decrease at identical rates, the X-force attrition coefficient can be written

$$\beta(t) = p_{ky}(mt + b). \tag{19}$$

The foregoing assumptions allowed the attrition coefficients to be expressed as a ratio of constants as in (16)

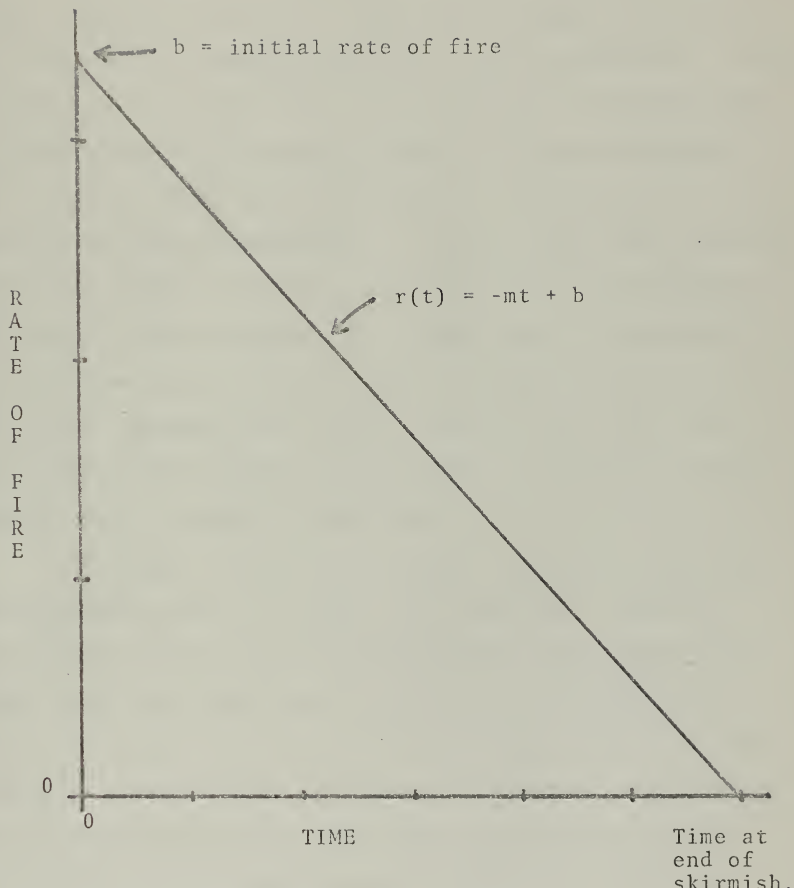

FIGURE THREE

RATE OF FIRE -- A LINEARLY DECREASING FUNCTION OF TIME

such that equations (14), (15), and (17) can be used to solve explicitly for X(t) and Y(t).

2. Numerical Computations

As with the previous, constant-coefficient models, a short computer routine was drawn up to compute attrition for a skirmish as modeled under the stated assumptions. Also as before, every effort was made to make the firefights modeled analogous in all respects, the only difference among them taking the form of a difference related directly to the specified different assumptions. In the case of the variable coefficient models, however, the restrictions encountered by the constant-ratio assumption described above necessitated consideration only of those models in which the rates of fire of the opponents were equal. Hence, force size and single-shot kill probability were the only parameters manipulated in the testing of this model.

In order to maintain tactical similarity among the previous models and the current time-dependent attrition model, force levels and kill probabilities were maintained at the identical values used in the constant-coefficient, square-law models described and tested above. The linearly decreasing rates of fire were set at an initial rate of fire of .334 rounds/second for both units, yielding an average rate of fire of .167 rounds/second, identical to the constant figure used in the previously discussed, square-law, constant-coefficient model.

3. Results and Conclusions

As with the previous comparison, attrition under the variable coefficient model yielded results that were somewhat

similar to the output of models tested earlier. Results are tabulated in Table Three and a typical case is graphed in Figure Four.

TABLE THREE

SQUARE LAW ATTRITION DATA -- LINEARLY DECREASING ATTRITION COEFFICIENTS

INITIAL STRENGTHS (MEN) X	INITIAL STRENGTHS (MEN) Y	RATE OF FIRE (rds/sec) X	RATE OF FIRE (rds/sec) Y	KILL PROBABILITIES X	KILL PROBABILITIES Y	TIME (sec)	INITIAL STRENGTHS (MEN) X	INITIAL STRENGTHS (MEN) Y
200	200	.278	.278	.001	.002	100	188	194
200	200	.223	.223	.001	.002	200	178	190
200	200	.167	.167	.001	.002	300	171	186
200	200	.111	.111	.001	.002	400	166	184
200	200	.056	.056	.001	.002	500	163	182
200	200	.000	.000	.001	.002	600	162	182
200	100	.278	.278	.001	.001	100	197	94
200	100	.223	.223	.001	.001	200	195	89
200	100	.167	.167	.001	.001	300	193	85
200	100	.111	.111	.001	.001	400	192	83
200	100	.056	.056	.001	.001	500	191	81
200	100	.000	.000	.001	.001	600	191	80
200	100	.278	.278	.001	.002	100	194	94
200	100	.223	.223	.001	.002	200	190	89
200	100	.167	.167	.001	.002	300	186	86
200	100	.111	.111	.001	.002	400	184	83
200	100	.056	.056	.001	.002	500	182	81
200	100	.000	.000	.001	.002	600	182	81

The most striking aspect of the variable-coefficient model was the shape of the attrition curves. While the order of magnitude of the attrition remained close to that of the analogous constant-coefficient model, the behavior of the attrition process is markedly different, being very definitely non-linear in shape.

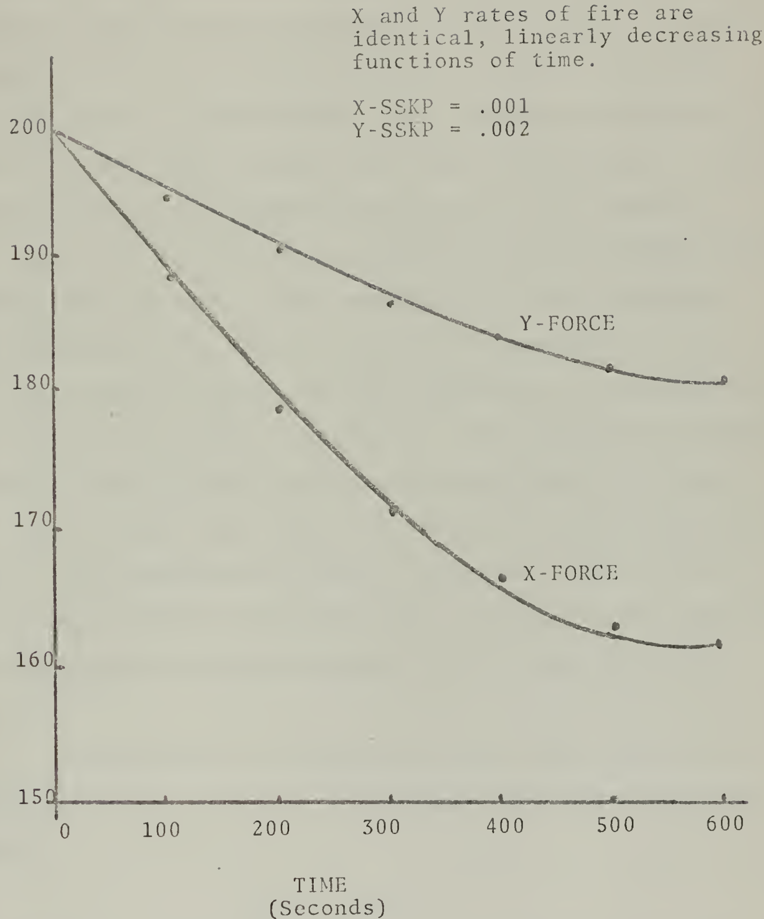

FIGURE FOUR

TYPICAL SQUARE-LAW ATTRITION -- VARIABLE COEFFICIENTS

B. EXPONENTIAL ATTRITION COEFFICIENTS

1. Scenario and Assumptions

A logical extension of the concept of linearly-decreasing attrition coefficients is that of the case where attrition rates decrease exponentially with time. In formulating attrition coefficients of such a nature, the author was still bound by the constant-ratio assumption discussed above. In this case, as with the linear coefficients, kill probabilities for the opposing forces were held constant, while the rates of fire were allowed to decrease (identically) exponentially to zero at the termination of the skirmish. Such behavior of the rate of fire in an engagement of the type being modeled was intuitively appealing to the author in the light of past combat experience. The exponential behavior of such a rate of fire is portrayed graphically in Figure Five. As in the linear case, rate of fire is quite high initially. With exponential behavior, however, rate of fire decreases very rapidly during the first seconds of the engagement, after which the decrease continues, but an ever-lessening rate.

Formulated in mathematical terms, this meant, for example, that the attrition coefficient $\alpha(t)$ would again be written

$$\alpha(t) = p_{ky} r(t) \ ,$$

where, however, a new function $r(t)$ was defined as follows:

$$r(t) = Re^{-\gamma t} \ . \tag{20}$$

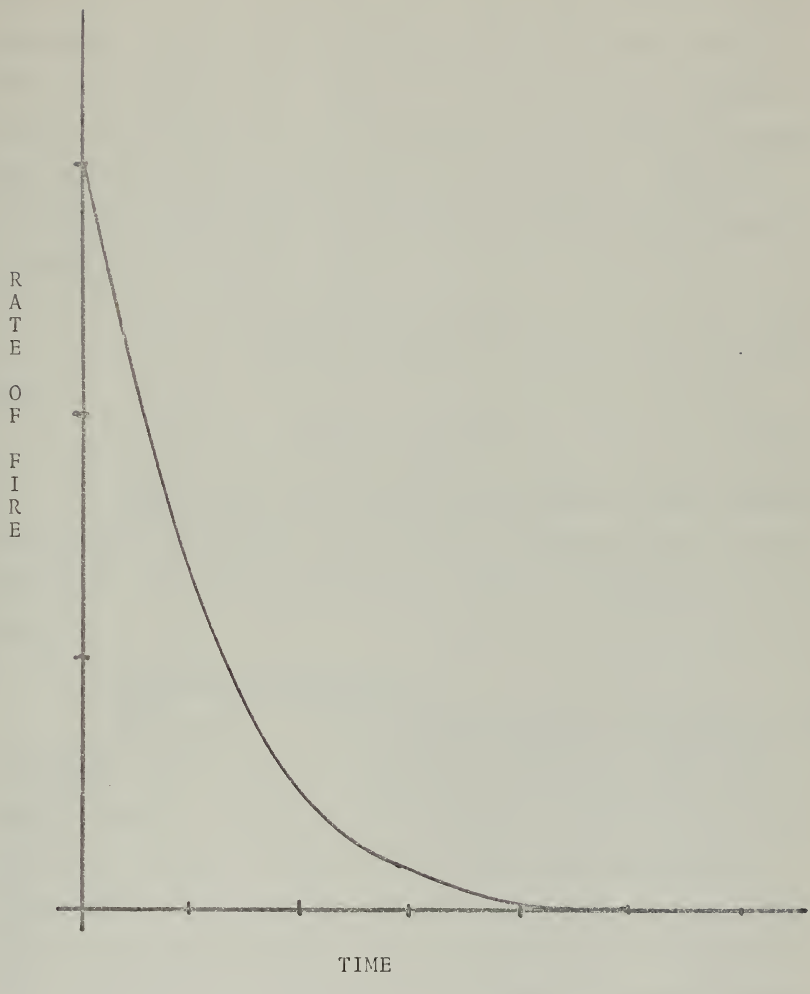

FIGURE FIVE

EXPONENTIALLY-DECREASING RATE OF FIRE

Values of R and γ must then be chosen such that the rate of fire varies between feasible limits for the weapons systems employed. It should be noted here that r(t), the rate of fire, is assumed to be identical for both forces, this because of the constant-ratio assumption, since r(t) is simply the function h(t) as written in equation (16). These considerations enable the two attrition coefficients to be expressed as

$$\alpha(t) = p_{ky}\ Re^{-\gamma t}\ , \tag{21}$$

and

$$\beta(t) = p_{ky}\ Re^{-\gamma t}\ . \tag{22}$$

These expressions allow maintenance of the constant ratio of the attrition coefficients, permitting the use of equations (14), (15), and (17) in explicit solutions as before.

2. Numerical Computations

Once again, a short computer program (appearing after the text) was drawn up to simulate a skirmish, using this attrition model. Insofar as was possible, the tactical situation as it existed in terms of the model was maintained identical to that described in previous models and only the rate of fire was changed. An approximation to the rate of fire used in the earlier square-law models was made, however, in that the average rate of fire in the exponential case was set close to the value of the average used in the linear attrition rate model, the latter, in turn, being equal to the rate of fire used in the constant-coefficient model. This

was accomplished by careful selection (via trial and error) of the parameters R and γ in equation (20).

3. Results and Conclusions

Attrition data from the exponential rate of fire model are printed in Table Four with typical attrition curves illustrated in Figure Six. Most noteable about the output from this model is the different attrition behavior exhibited. By far the heaviest casualties suffered by both sides are inflicted during the earliest stages of the engagement, with relatively few -- if any -- casualties being suffered during the final minute or two of the skirmish. This aspect in particular seems to conform more closely to reality with the tendency of most such conflicts to begin sharply and fade out gradually.

TABLE FOUR

SQUARE LAW ATTRITION DATA -- EXPONENTIALLY DECREASING ATTRITION COEFFICIENTS

INITIAL STRENGTHS (MEN)		RATES OF FIRE (rds/sec)		KILL PROBABILITIES		TIME	FINAL STRENGTHS (MEN)	
X	Y	X	Y	X	Y	(sec)	X	Y
200	200	.368	.368	.001	.002	100	176	188
200	200	.135	.135	.001	.002	200	167	184
200	200	.050	.050	.001	.002	300	164	183
200	200	.018	.018	.001	.002	400	162	182
200	200	.007	.007	.001	.002	500	162	182
200	200	.002	.002	.001	.002	600	162	162
200	100	.368	.368	.001	.001	100	188	88
200	100	.135	.135	.001	.001	200	184	83
200	100	.050	.050	.001	.001	300	183	82
200	100	.018	.018	.001	.001	400	182	81
200	100	.007	.007	.001	.001	500	182	81
200	100	.002	.002	.001	.001	600	182	81

200	100	.368	.368	.001	.002	100	194	88
200	100	.135	.135	.001	.002	200	192	83
200	100	.050	.050	.001	.002	300	191	81
200	100	.018	.018	.001	.002	400	191	81
200	100	.007	.007	.001	.002	500	191	81
200	100	.002	.002	.001	.002	600	191	80

TABLE FOUR Continued

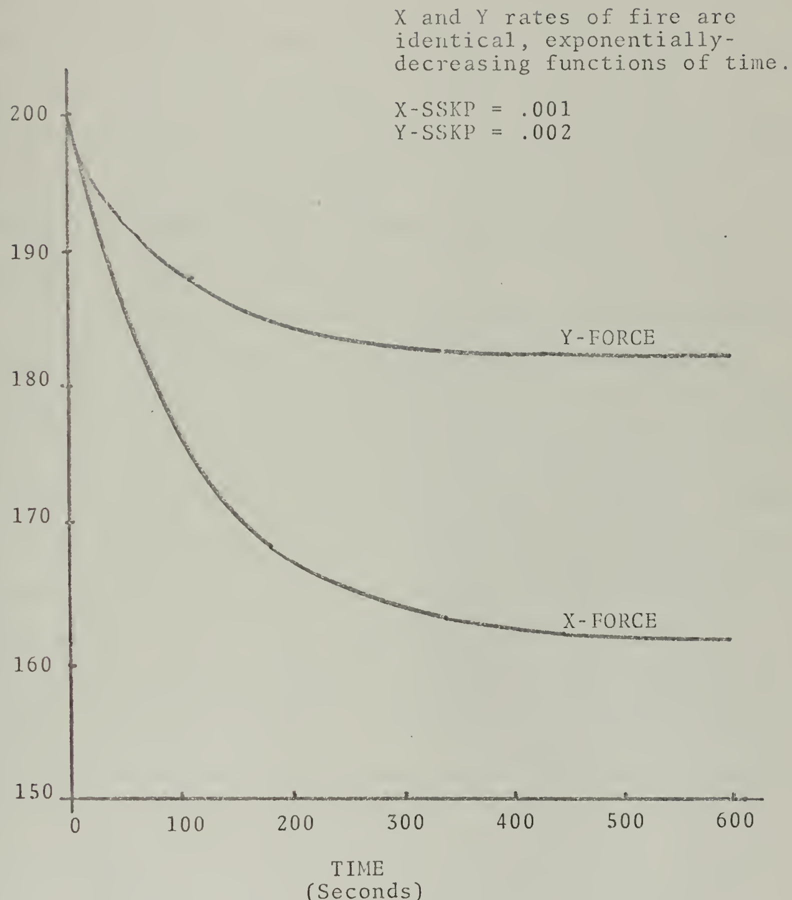

FIGURE SIX

TYPICAL SQUARE LAW ATTRITION -- VARIABLE COEFFICIENTS

V. CONCLUSIONS AND RECOMMENDATIONS

Time-dependent attrition coefficients appear to be a definite improvement over constant coefficients for use in realistic Lanchester models of ground combat. This holds true even under the apparently difficult restriction requiring the attrition coefficients to have a constant ratio, the latter restriction imposed as a means of obtaining a simple and readily useable closed-form solution to the Lanchester square law equations.

In modeling the skirmish, attrition coefficients which decreased exponentially with time appeared to cause the square law model to yield the most realistic attrition process behavior, although the same model employing linearly-decreasing coefficients also yielded very plausible results. Constant-coefficient models of both the square law and the linear law cases yielded data that were almost identical for both models. While this trend would not be expected to hold up for more extended periods of combat, it did appear that either model would suffice to describe the skirmish, insofar that the attrition figures were fairly close to those yielded by the variable-coefficient models. On the other hand, the latter models did yield attrition processes that were certainly more appealing from the standpoint of military experience.

Even within the constant-ratio restriction, there is considerable room for more analysis. Allowing kill probabilities to vary (as well as rates of fire) and testing

the square-law model with the resulting attrition coefficient functions would produce added insight. Focusing attention on the more general problem, wherein no restrictions are placed on the attrition function could prove fruitful, although it appears that preliminary work in this direction has resulted in closed-form solutions that are highly complicated. However, the testing of certain numerical solutions and the use of finite difference approximations would certainly appear to be in order.

Extending the concept of variable coefficients to the case of the area-fire, linear-law model should also result in added insight and a wider applicability of this aspect of the Lanchester theory of combat. Again, where explicit analytical solutions are unavailable or extremely complicated, numerical solutions could be used in investigating the attrition processes described by such models.

The Lanchester theory of combat enjoys wide popularity as a tool of practical workers as well as theoretical researchers. Its use can be extended even more via the employment of variable attrition coefficients which allow the Lanchester models to better reflect the whims and vagaries of actual combat attrition. As the military services turn more and more to simulation models and computerized war games as sources of data upon which to base future force levels and procurement goals, the use of wider variety of Lanchester models which reflect the combat situation more realistically becomes desirable. In the forefront of such an expanded class of models are those with time-dependent attrition rates.

```
      SQUARE LAW ATTRITION -- CONSTANT COEFFICIENTS

           THIS PROGRAM COMPUTES COMBAT ATTRITION VIA THE
       LANCHESTER SQUARE LAW.  THE MAIN PROGRAM ACTS TO
       ASSIGN PARAMETER VALUES AND THEN CALLS SUBROUTINE
       "SQUARE" WHICH PERFORMS THE ACTUAL COMPUTATIONS.
      X$ONE  = 2   .
      Y$ONE=2   .
      XRAT1=.167
      YRAT1=.167
      PX1=.  1
      PY1=.  2
      WRITE (6,  1)
    1 FORMAT('1',37X,'TABLE 1: SQUARE LAW ATTRITION DATA --
     1CONSTANT  COEFFICIENTS.)
      WRITE(6,2  1)
 2  1 FORMAT(' ','X-ORIG',  X,'Y-ORIG',8X,'X-RATE/FIRE',8X,'Y
     1-RATE/FIRE',  X,'SSHP-X',  X,'SSHP-Y',  X,'TIME(SEC)',3X,
     2'X(T)',  X,'Y(T)')
      CALL SQUARE(X$ONE,Y$ONE,XRAT1,YRAT1,PX1,PY1)
      X$TWO=2    .
      Y$TWO=2    .
      XRAT2=.167
      YRAT2=. 84
      PX2=.  1
      PY2=.  1
      CALL SQUARE(X$TWO,Y$TWO,XRAT2,YRAT2,PX2,PY2)
      XTHRE=2    .
      YTHRE=1    .
      XRAT3=.167
      YRAT3=.167
      PX3=.  1
      PY3=.  1
      CALL SQUARE(XTHRE,YTHRE,XRAT3,YRAT3,PX3,PY3)
      XFOUR=2    .
      YFOUR=1    .
      XRAT4=.  4
      YRAT4=.167
      PX4=.  1
      PY4=.  1
      CALL SQUARE(XFOUR,YFOUR,XRAT4,YRAT4,PX4,PY4)
      XFIVE=2    .
      YFIVE=1    .
      XRAT5=. 84
      YRAT5=.167
      PX5=.  1
      PY5=.  2
      CALL SQUARE(XFIVE,YFIVE,XRAT5,YRAT5,PX5,PY5)
      STOP
      END


      SUBROUTINE SQUARE(XINIT,YINIT,XRATE,YRATE,PKX,PKY)

           THIS SUBROUTINE PERFORMS THE ACTUAL ATTRITION
       COMPUTATIONS FOR THOSE PARAMETER VALUES ASSIGNED BY
       THE CALLING PROGRAM.

      DIMENSION XT(6),YT(6)
      ALPHA = YRATE PKY
      BETA = XRATE PKX
      ARG1=ALPHA BETA
      ARG2=ALPHA/BETA
      ARG3 = BETA/ALPHA
      ROOT1=SQRT(ARG1)
      ROOT2=SQRT(ARG2)
      ROOT3=SQRT(ARG3)
      DO 1   I=1,6
      ITIME=I 1
      THETA = ROOT1 TIME
      FAC1=COSH(THETA)
```

```
      FAC2=SINH(THETA)
      XT(I) = XINIT FAC1-YINIT ROOT2 FAC2
      YT(I)=YINIT FAC1-XINIT ROOT3 FAC2
      WRITE(6,2  2)XINIT,YINIT,XRATE,YRATE,PKX,PKY,ITIME,
     1XT(I),YT(I)
 2  2 FORMAT(' ',1X,F4. ,1 X,F4. ,11X,F6.5,13X,F6.5,12X,F4.
     15,1 X,F .3,12X,I3,11X,F5.1,7X,F5.1)
 1    CONTINUE
      RETURN
      END
```

LINEAR LAW -- CONSTANT COEFFICIENTS

THIS PROGRAM DETERMINES COMBAT ATTRITION UNDER THE LANCHESTER LINEAR LAW. THE STRUCTURE OF THE SIMULATION IS SUCH THAT THE CALLING PROGRAM MERELY ASSIGNS PARAMETER VALUES, AFTER WHICH IT SUMMONS

SUBROUTINE "LINEAR" TO PERFORM THE ATTRITION COMPUTATIONS.

```
      COMMON/AREA/AEX,AEY
      X$ONE = 2  .
      Y$ONE = 2  .
      XRAT1=.334
      YRAT1=.334
      AEX=1.
      AEY=1.
      AX1=2      .
      AY1=       .
      WRITE(6,3  1)
 3  1 FORMAT('1',  X,'TABLE 2: LINEAR LAW ATTRITION DATA ')
      WRITE(6,3  2)
 3  2 FORMAT(' ',  X,'PHI',  X,'X-ORIG',7X,'Y-ORIG',5X,'RATE/
     1FIRE-X',4X,'RATE/FIRE-Y',3X,'AREA/Y-FORCE',3X,'AREA/X-F
     2ORCE',5X,'TIME',  X,'X(T)',  X,'Y(T)')
      CALL LINEAR(X$ONE,Y$ONE,XRAT1,YRAT1,AX1,AY1)
      X$TWO=2  .
      Y$TWO=2  .
      XRAT2=.334
      YRAT2=.167
      AX2=4      .
      AY2=4      .
      CALL LINEAR(X$TWO,Y$TWO,XRAT2,YRAT2,AX2,AY2)
      XTHRE=2  .
      YTHRE= 1  .
      XRAT3=.334
      YRAT3=.334
      AX3=4      .
      AY3=2      .
      CALL LINEAR(XTHRE,YTHRE,XRAT3,YRAT3,AX3,AY3)
      XFOUR=2  .
      YFOUR=1  .
      XRAT4=.167
      YRAT4=.334
      AX4=4      .
      AY4=2      .
      CALL LINEAR(XFOUR,YFOUR,XRAT4,YRAT4,AX4,AY4)
      XFIVE = 199.
      YFIVE=1  .
      XRAT5=.167
      YRAT5=.334
      AX5=2      .
      AY5=2      .
      CALL LINEAR(XFIVE,YFIVE,XRAT5,YRAT5,AX5,AY5)
      STOP
      END
```

```
      SUBROUTINE LINEAR(XINIT,YINIT,XRATE,YRATE,AX$,AY$)

           THIS SUBROUTINE PERFORMS THE ACTUAL COMPUTATIONS
       DETERMINING THE ATTRITION FIGURES ACCORDING TO THE
       LANCHESTER LINEAR LAW. THE SKIRMISH MODELED IS ASSUM
       ING AREA FIRE.

      COMMON/AREA/AEX,AEY
      ALPHA=YRATE AEX/AX$
      BETA=XRATE AEY/AY$
      PHI=(BETA XINIT)/(ALPHA YINIT)
      RHO=PHI-1.
      XFAC=-XINIT RHO
      YFAC=-YINIT RHO
      DO 100 I=1,6
      ITIME=100 I
      ARG=-ALPHA YINIT RHO ITIME
      FAC=EXP(ARG)
      DENOM=FAC-PHI
      XT=(-XINIT RHO)/DENOM
      YT=(-YINIT RHO FAC)/DENOM
      WRITE(6,2001)PHI,XINIT,YINIT,XRATE,YRATE,AY$,AX$,ITIME
     1,XT,YT
 2001 FORMAT(' ',F11.6,5X,F5.1,8X,F5.1,9X,F5.3,10X,F5.3,9X,F
     17.0,10X,F7.0,10X,I3,4X,F6.1,6X,F6.1)
  100 CONTINUE
      RETURN
      END
```

SQUARE LAW -- LINEAR COEFFICIENTS

```
           THIS PROGRAM DETERMINES COMBAT ATTRITION
       USING LANCHESTER'S SQUARE LAW. ATTRITION COEF-
       FICIENTS ARE VARIABLE FUNCTIONS OF TIME, DEPENDING
       UPON A CONSTANT SINGLE-SHOT KILL PROBABILITY AND A
       DECREASING RATE OF FIRE.
           THE PROGRAM OPERATES BY USING THE MAIN PROGRAM
       AS A MEANS FOR SETTING PARAMETER VALUES: IT THEN
       CALLS SUBROUTINE "VARLIN" WHICH PERFORMS THE ACTUAL
       COMPUTATIONS, PRINTING OUT SIGNIFICANT COMBAT DATA
      EVERY 100 SECONDS, FOR AN ENGAGEMENT LASTING 600 SEC.

      X$ONE=200.
      Y$ONE=200.
      XRAT1=.334
      YRAT1=.334
      PX1=.01
      PY1=.02
      ITIME=600
      WRITE(6,3001)
 3001 FORMAT('1',32X,'TABLE 3: SQUARE LAW ATTRITION DATA --
     1VARIABLE COEFFICIENTS (LINEAR)',/,'+',32X,'_____ __ ___
     2___ ___ _________ ____       ________ ____________  _____
     3__.')
      WRITE(6,3002)
 3002 FORMAT('0','X-ORIG',8X,'Y-ORIG',8X,'X-RATE/FIRE',8X,
     1'Y-RATE/FIRE',8X,'SSKP-X',8X,'SSKP-Y',8X,'TIME(SEC)'
     2,6X,'X(T)',8X,'Y(T)')
      CALL VARLIN(X$ONE,Y$ONE,XRAT1,YRAT1,PX1,PY1,ITIME)
      Y$TWO=100.
      PY2=.01
      CALL VARLIN(X$ONE,Y$TWO,XRAT1,YRAT1,PX1,PY2,ITIME)
      CALL VARLIN(X$ONE,Y$TWO,XRAT1,YRAT1,PX1,PY1,ITIME)
      STOP
      END
```

```
      SUBROUTINE VARLIN(XINIT,YINIT,XRATE,YRATE,PKX,PKY,ITIM
     1E)
C
C         THIS SUBROUTINE COMPUTES THE EXPLICIT ATTRITION
C      FIGURES UNDER THE LANCHESTER SQUARE LAW WITH VARIABLE
C      COEFFICIENTS.
C
      FAC1=PKX PKY
      FAC2=PKX/PKY
      FAC3=PKY/PKX
      SROOT1=SQRT(FAC1)
      SROOT2=SQRT(FAC2)
      SROOT3 = SQRT(FAC3)
      SLOPE = -XRATE/ITIME
      ARG1 = SLOPE/2.
      X1 = XINIT
      Y1 = YINIT
      DO 1   I = 1  ,ITIME,1
      HINT = ARG1 I  2+XRATE I
      THETA = -SROOT1 HINT
      BALL = COSH(THETA)
      STRIK = SINH(THETA)
      XT = X1 BALL+Y1 SROOT3 STRIK
      YT = Y1 BALL+X1 SROOT2 STRIK
      RATX=SLOP  I+XRATE
      WRITE (6,2  1)XINIT,YINIT,RATX,RATX,PKX,PKY,I,XT,YT
2  1  FORMAT(' ',1X,F6. ,11X,F6. ,13X,F6.5,12X,F6.3,1 X,F6.3
     1,12X,I3,11X,F5.1,7X,F5.1)
1     CONTINUE
      RETURN
      END
C
C              SQUARE LAW -- EXPONENTIAL COEFFICIENTS
C
C         THIS PROGRAM COMPUTES COMBAT ATTRITION VIA THE
C      LANCHESTER SQUARE LAW WHERE ATTRITION COEFFICIENTS
C      ARE DECREASING EXPONENTIAL FUNCTIONS OF TIME.  AFTER
C      ASSIGNING COMBAT PARAMETERS, THE MAIN PROGRAM CALLS
C
C
C      SUBROUTINE "VAREX" WHICH PERFORMS THE ACTUAL COMPU-
C      TATIONS.
C
      X$ONE=2 . .
      Y$ONE=2   .
      PX1=.  1
      PY1=.  2
      ITIME = 6
      WRITE(6,3  1)
3  1  FORMAT('1',27X,'TABLE 4: SQUARE LAW ATTRITION DATA --
     1VARIABLE COEFFICIENTS(EXPONENTIAL)')
      WRITE(6,3  2)
3  2  FORMAT(' ','X-ORIG',5X,'Y-ORIG',8X,'X-RATE/FIRE',8X,
     1'Y-RATE/FIRE',8X,'SSKP-X',8X,'SSKP-Y',8X,'X(T)',8X,'Y(
     2T)')
      CALL VAREX(X$ONE,Y$ONE,PX1,PY1,ITIME)
      Y$TWO=1   .
      CALL VAREX(X$ONE,Y$TWO,PX1,PY1,ITIME)
      PY3=.  1
      CALL VAREX(X$ONE,Y$TWO,PX1,PY3,ITIME)
      STOP
      END
```

```
      SUBROUTINE VAREX(XINIT,YINIT,PKX,PKY,ITIME)

      FAC1 = PKX PKY
      FAC2=PKX/PKY
      FAC3=1./FAC2
      SROOT1=SQRT(FAC1)
      SROOT2=SQRT(FAC2)
      SROOT3=SQRT(FAC3)
      DO 1[illegible] I = 1[illegible],6[illegible],1[illegible]
      ARG=-.[illegible]1 I
      ARG1 = EXP(ARG)
      HINIT = 1[illegible]. - 1[illegible]. ARG1
      THETA=-SROOT1 HINIT
      BALL = COSH(THETA)
      STRIK=SINH(THETA)
      XT=XINIT BALL+YINIT SROOT3 STRIK
      YT=YINIT BALL+XINIT SROOT2 STRIK
      RATXY = ARG1
      WRITE(6,2[illegible]1)XINIT,YINIT,RATXY,RATXY,PKX,PKY,I,XT,YT
2[illegible]1 FORMAT(' ',1X,F[illegible].[illegible],1[illegible]X,F4.[illegible],1[illegible]X,F7.5,12X,F7.5,12X,F4.3
     112X,I?,[illegible]X,F7.1,5X,F7.1)
1[illegible] CONTINUE
      RETURN
      END
```

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13. ABSTRACT

Four Lanchester-type models are examined to investigate the hypothetical attrition process in skirmishes between ground forces. Analytic solutions are developed to Lanchester-type equations of warfare for combat between two homogeneous forces in the following circumstances:

(1.) linear-law attrition process,

(2.) square-law attrition process with constant attrition-rate coefficients,

(3.) square-law attrition process with linearly-describing time varying attrition rates,

(4.) square-law attrition process with exponentially-decreasing, time varying attrition rates.

The above models are applied to specific combat scenarios typical of a counterinsurgency environment. The adequacy of such models as defense planning guides is discussed through a critical examination of the assumptions (both explicit and implicit) which lead to them.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Attrition Coefficient						
Firefight						
Lanchester						
Linear Law						
Model						
Simulation						
Skirmish						
Square Law						
War Game						

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